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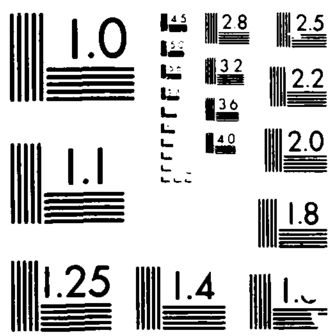

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) A useful function for analyzing burn-in, developing maintenance policies, or simply modeling lifetimes of equipment is the mean residual life function. Other functions, such as the reliability or the failure rate functions, are of course important also. Discrete data arises naturally in various ways: from discretizing or grouping continuous data, devices operate by "cycles" (e.g., a copier's "cycle" is a copy, its "lifelength" the total number of copies), etc. This paper develops a general approach to modeling discrete bathtub and upside down bathtub mean residual life functions. Because the approach allows parametric modeling of the mean residual life, maximum likelihood estimation of models can be done. This will enable estimation of such parametric models for complete discrete data, as well as right censored discrete data. A simple, perhaps surprising, example is presented where the mean residual life increases, then decreases; however, the hazard rate also increases, drops suddenly at one cycle, then increases again. ~~We~~ (The authors) discuss two reasonable industrial explanations of such unusual behavior.

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Key Words - Discrete mean residual life function, Discrete failure (hazard) rate function, Bathtub and upside down bathtub mean residual life, Modeling, Discrete life (failure) data, Maximizing reliability.

Abstract - A useful function for analyzing burn-in, developing maintenance policies, or simply modeling lifetimes of equipment is the mean residual life function. Other functions, such as the reliability or the failure rate functions, are of course important also. Discrete data arises naturally in various ways: from discretizing or grouping continuous data, devices operate by "cycles" (e.g., a copier's "cycle" is a copy, its "lifelength" the total number of copies), etc. This paper develops a general approach to modeling discrete bathtub and upside down bathtub mean residual life functions. Because the approach allows parametric modeling of the mean residual life, maximum likelihood estimation of models can be done. This will enable estimation of such parametric models for complete discrete data, as well as right censored discrete data. A simple, perhaps surprising, example is presented where the mean residual life increases, then decreases; however, the hazard rate also increases, drops suddenly at one cycle, then increases again. We discuss two reasonable industrial explanations of such unusual behavior.

1. INTRODUCTION

Modeling of the aging process of components and equipment can be performed in various ways. One helpful tool for that modeling, as well as for analyzing burn-in or developing maintenance policies, is the mean residual life function. Of course, other functions, such as the reliability or the failure rate functions, are important also.

An introduction to MRL (this and other notations are defined in Section 2) and its many applications are surveyed in Guess and Proschan [4]. See also Bhattacharjee [2] and Gupta [6]. Kuo [9] presented a review in his Appendix 1 on burn-in and MRL. Park [10] commented, "... the time at which a bathtub failure-rate is a minimum does not maximize the mean residual life. Contrary to popular belief, the mean residual life in the constant failure-rate region of a bathtub failure-rate curve is not constant." Of course, this and other MRL facts have important consequences for developing better burn-in policies.

Discrete data arises naturally in various situations: from discretizing or grouping of continuous data, devices operate by "cycles," etc. For example, a copier and its key components have as their "cycle" producing a copy. The lifelength would be the total number of copies.

Recently, Ebrahimi [3] has proposed models for discrete DMRL and IMRL. He also referenced earlier work on other types of discrete failure models for life testing data.

Monotone aging models are very useful and important in reliability applications. E.g., a Weibull with a shape parameter greater than 1 is a DMRL (also an IFR) model - adverse aging. A Weibull with a shape parameter less than 1 (and greater than 0, of course) is an IMRL (also a DFR) model - beneficial aging. Recall IFR implies DMRL, while DFR implies IMRL. On the other hand, models with more than one stage of aging are also important and useful in reliability practice. E.g., the empirically observed bathtub failure rate model has aging stages of infant mortality, useful life, and wearout. See Barlow & Proschan [1, pp.55-56] for an example on a commercial airline engine's hot gas generating subsystem. We use the same convention as Barlow & Proschan [1, p.6] and others of using "increasing" for "nondecreasing" (constant is allowed) and "decreasing" for "nonincreasing."

Note that bathtub failure rate models are in the nonparametric class of DIFR. The related class to DIFR is IDMRL. The lognormal, used for repair times as well as lifetimes, is in the IDFR class. The analog to IDFR is DIMRL. See Guess, Hollander & Proschan [5] for details on the IDMRL and DIMRL classes.

An upside down bathtub MRL is in the IDMRL class (human lifelength can be modeled well by this class). A bathtub MRL is in the DIMRL class (repair time models are included).

In this paper, we consider discrete versions of IDMRL and DIMRL. A general approach to creating parametric and nonparametric models in these classes is presented. Illustrative parametric examples are given. This example includes as a special case one of Ebrahimi's [3] model by allowing $n_0 = 0$.

Because the approach allows parametric modeling, maximum likelihood estimation of the models can be done. This estimation would be possible, not only for complete discrete life data, but even for right censored discrete data. Cf. Hall & Wellner [7].

A simple, perhaps surprising, example is presented of IDMRL but not DIFR. In fact, the failure rate and the MRL both increase over the set $n=0,1,\ldots, n_0-1$

of cycles. At the turning point, n_0, from IMRL to DMRL the failure rate drops suddenly and increases again. See Figures 1 and 2. We discuss two possible industrial explanations of such unusual behavior. Another simple, interesting example shows that even if a distribution is DIFR and IDMRL the turning points in each can differ. See Section 3.

2. NOTATION & NOMENCLATURE

Notation

T	time to failure of a system (or component)
f_i	$pmf\{t_i\} = Pr\{T=t_i\}$
R_i	$Sf\{t_i\} = Pr\{T \geq t_i\}$
$R(x)$	$Sf\{x\} = Pr\{T \geq x\}$
λ_i	discrete failure rate; f_i/R_i
$\bar{\lambda}_i$	complement of the discrete failure rate; $\bar{\lambda}_i \equiv 1-\lambda_i$
m_i	mean residual life; $E[T-t_i \mid T \geq t_i]$
MRL	mean residual life
DMRL	decreasing MRL
IMRL	increasing MRL
IDMRL	increasing initially then decreasing MRL
DIMRL	decreasing initially then increasing MRL
IFR	increasing failure rate
DFR	decreasing failure rate
DIFR	decreasing initially then increasing failure rate
IDFR	increasing initially then decreasing failure rate

Nomenclature

IDMRL: A life distribution is called IDMRL at n_0 if $m_i \leq m_{i+1}$ for

$i=0,1,\ldots, n_0-1$ and $m_i \geq m_{i+1}$ for $i=n_0, n_0+1,\ldots$. Note that the MRL is maximized at time n_0, the "turning point."

DIRML: A life distribution is called DIMRL at n_0 if $m_i \geq m_{i+1}$ for $i=0,1,\ldots,n_0-1$ and $m_i \leq m_{i+1}$ for $i=n_0,n_0+1,\ldots$. Note that the MRL is minimized at time n_0, the "turning point."

3. A GENERAL APPROACH FOR CREATING IDMRL AND DIMRL MODELS

Consider again the definition of the MRL function

$$m_i = E[T-t_i \mid T\geq t_i]. \tag{2.1}$$

This can be rewritten as

$$m_i = \int_{t_i}^{\infty} R(x)dx/R_i \tag{2.2}$$

Cf., for example, Kotz & Shanbhag [8]. When continuous data is grouped or when it is only possible to observe failure that occurred in some interval, the t_i's could have a fractional part and/or be spaced unevenly. (E.g., during night shifts and weekends, the inspection times for failure of a device can be less frequent and sometimes uneven.) Allowing for this and assuming the t_i's are ordered ($t_0 < t_1 < \ldots$), we can express (2.2) as

$$\begin{aligned} m_i &= \left(\sum_{j=i}^{\infty} \int_{t_j}^{t_{j+1}} R(x)dx \right) /R_i \\ &= \left(\sum_{j=i}^{\infty} (t_{j+1} - t_j)\ R(t_{j+1})\right) / R_i \\ &\equiv \left(\sum_{j=i}^{\infty} (t_{j+1} - t_j)R_{j+1}\right) /R_i \end{aligned} \tag{2.3}$$

Recall in the above that R(x) is left continuous, not right continuous.

To allow for fractional values and uneven spacings, a person would want to use (2.3). On the other hand, for the case $t_{j+1} - t_j = 1$, typical of devices (e.g., copiers) with cycles, (2.3) has the simple form of

$$m_i = \sum_{j=i}^{\infty} R_{j+1} / R_i \tag{2.4}$$

A simple quick check of (2.4) is to take T degenerate 0. (Note that $R_0=1$, $R_i=0$ for $i \geq 1$; thus, $m_0=R_1/R_0 + 0 + 0 + \ldots = 0$. $m_i = 0$ for $i \geq 1$ by convention).

Because we have been motivated primarily by devices with cycles and previous authors have focused on the case of $t_j = j$ for $j = 0,1,2,\ldots$, we consider (2.4) for the rest of this paper. We also assume $m_i > 0$ for $i=0,1,2,\ldots$ in all of the following.

The main tool in our method for producing IDMRL and DIRML models is the following theorem. See the Appendix for its proof.

Theorem 1. A life distribution is IDMRL at n_0 if and only if there exist a sequence $\{a_n\}$, $a_n > 0$ for all n such that:

$$\left.\begin{array}{l} a_n \leq a_{n+1} \text{ for } n=0,1,\ldots,n_0-1 \\ a_n \geq a_{n+1} \text{ for } n=n_0,n_0,+1,\ldots \end{array}\right\} \tag{2.5}$$

and $\bar{\lambda}_n = a_n/(1+a_{n+1})$ for $n=0,1,2,\ldots$. (2.6)

A similar result for a DIMRL at n_0 distribution is obtained by reversing the inequalities in (2.5). See the end of the Appendix for further comments.

The proof in the Appendix demonstrates the helpful insight that $a_n \equiv m_n$. This makes building models even easier. In creating an IDMRL at n_0 model we simply need to generate a sequence with $a_n > 0$ satisfying (2.5) and (2.6). Note that condition (2.5) is not difficult to meet. In the case of checking whether a known distribution is IDMRL at n_0, then (2.6) is also straightforward. Condition (2.6) may seem rather awkward, however, to verify in the situaton of creating a new distributional model. Instead of verifying (2.6) directly, it suffices to determine that

$$\sum_{n=0}^{\infty} \frac{1}{a_n} = \infty ,$$

$$a_{n+1} - a_n \geq -1 , \ n=0,1,\ldots$$

and, of course, $a_n > 0$ for all n. When a sequence is increasing after some point, the above inequality is always met, while the equality may or may not hold. When a sequence decreases anywhere, the inequality implies the decrease can not be too quickly. When a sequence is decreasing after some time, the equality will always hold. (Aside: it turns out these alternative conditions are necessary ones and can not be dropped. They will always be implicit in other conditions. Cf. Bhattacherjee [2] and Hall & Wellner [7].)

Although the increase (decrease) of the MRL might suggest the decrease (increase) of the hazard rate of the life distribution, the following example shows that the turning point for IDMRL does not necessarily agree with the turning point from decreasing hazard rate to increasing hazard rate. Compare also Example 2.

<u>Example 1.</u> Let $a_o = 1$, $a_1 = 3/2$, $a_2 = 4/3$, $a_3 = 1$ and $a_n = 1/3$ for $n \geq 4$. Then it is easy to verify that the turning point from DFR to IFR occurs at n=3. However, according to Theorem 1, the turning point for IDMRL occurs at n_0=1.

Ebrahimi [3] considered the following illustrative discrete DMRL model with

$$a_n \equiv c/(\alpha n+1), \ n=0,1,2,\ldots, \ (0\leq c\leq 1, \ \alpha\geq 0). \tag{2.7}$$

It can be shown directly that $a_n \equiv m_n < 1$ for n=0,1,2,... for model (2.7). Note that $m_n < 1$ is a severe restriction, of course. A very slight, natural modification of the model, however, is possible. Let T be a random variable with distribution formed from the $\{a_n\}$ of (2.7). The sequence $\{a_n\}$ that generates the distribution of $T' = \theta T$ for $\theta>0$ is

$$a_n = \theta^2 c/(\alpha n+\theta),\ n=0,1,2,\ldots,\ (0\leq c\leq 1,\ \alpha\geq 0,\theta>0). \qquad (2.8)$$

Note that $E(T') = \theta\cdot E(T)$ and that for $\theta=1$ (2.8) reduces to (2.7). Also recall $m_n \equiv a_n$.

To get a simple discrete IDMRL model, we let

$$a_n = \theta^2 c/(\alpha|n-n_0| + \theta)$$

for $n=0,1,2,\ldots$, $(0\leq c\leq 1,\ \alpha\geq 0,\ \theta>0, n_0\geq 0)$. An easy check of Theorem 1 shows the above valid. This model includes both (2.7) and (2.8) as special cases. (The case $n_0=0$ is allowed.)

It is possible to simplify this IDMRL a_n by letting $c = 1$; thus,

$$a_n = \theta^2 / (\alpha\ |\ n-n_0| + \theta).$$

Note that $\lim_{n\to\infty} a_n = 0$ is a natural condition on the MRL. Recall the failure rate of the Gamma life distribution, however, converges to a constant greater than 0 as time t gets larger. I.e., asymptotically in time it behaves like an exponential. If a situation suggests that $\lim_{n\to\infty} a_n = a > 0$ is needed on the MRL, another possible modification is

$$a_n = \theta^2/(\alpha|n-n_0| + \theta) + \gamma,\ n=0,1,2,\ldots\ , \qquad (2.9)$$

with the new parameter $\gamma \geq 0$. If a reliability analyst wanted the essential flexibility of (2.9), yet a simplified version then just set $\alpha = 1$.

Example 2. Let $\alpha = 1$, $n_0 = 40$, $\theta = 62.711168$, and $\gamma = 61.711168$ in (2.9). To get a mean (m_0) of 100, we used this θ and γ. The MRL function, m_i, is graphed in Figure 1. Note that the graph is as expected IDMRL at $n_0 = 40$. Figure 2 shows the graph of the failure rate function. Note that it is not, however, DIFR. In fact, both the failure rate and the MRL strictly increase for n=0, 1,..., $39=n_0-1$. (Recall IFR everywhere implies DMRL everywhere.) At the turning point ($n_0 = 40$) from IMRL to DMRL the failure rate drops suddenly to its minimum and then increases again.

One possible explanation for the unusual behavior is that at the turning point an overhaul is performed with new, state of the art parts. The higher initial IFR is replaced by a lower IFR immediately upon completion of the overhaul. The closer in time the device can get to the overhaul time and the significantly lower failure rate, the larger the MRL grows. This explains the initial IMRL part, even though the failure rate is increasing. In the tail ($n \geq n_0 = 40$) we have IFR, thus DMRL.

Another explanation is to consider a stress screening program of parts or devices with IFR. In the initial time period the stressful screen results in a much higher than normal IFR. This is followed upon completion by a sudden drop to a significantly lower IFR. The closer we get to the time of exiting the screen, the higher the MRL gets, i.e., we have IMRL initially. After exiting to the lower IFR we have DMRL.

Figures 1 and 2 also show the importance of graphing both the MRL and the failure rate to understand data better. It will not necessarily be obvious looking at one graph what the other does.

4. CONCLUSIONS

For cost analysis, burn-in, and/or modeling discrete life data, the MRL is a useful tool. We have provided a general approach for creating IDMRL (e.g., useful for lifetimes) and DIMRL (e.g., useful for repair times) models.

Given a parametric version of one of these models, it is possible using (2.6) to derive the likelihood for the data. See Ebrahimi [3] and his references. Cf. Hall and Wellner [7]. With the likelihood it becomes possible to do maximum likelihood estimation on complete discrete life data and even on right (and/or left) censored data.

An advantage to parametrizing directly the MRL as presented in this paper is that even given the failure rate or the probability mass function, calculation of the MRL could require doing an infinite summation numerically. Cf. (2.4) and (A.1).

It should be noted again that minimizing the failure rate does not guarantee maximizing the MRL. Which measure of reliability does a person need "maximized" is the key question. Although for many situations minimizing the failure rate is best, for many others maximizing the MRL would be preferable because of reducing the average cost, for instance. (Cf. Kuo [9] and Guess and Proschan [4]). Our approach to constructing discrete IDMRL models contributes there also.

We presented illustrative examples and models to demostrate key advantages of thinking directly in terms of discrete MRL models. The graphs in Figures 1 and 2 showed that merely graphing the failure rate (MRL) alone will not necessarily suggest the actual behavior of the MRL (failure rate). Both are needed.

APPENDIX

This appendix provides the proof of Theorem 1. Salvia and Bollinger [11] gave representation results which we use below to obtain

$$f_0 = \lambda_0 = 1 - a_0/(1+a_1)$$

$$f_i = \lambda_i \prod_{j=0}^{i-1} \bar{\lambda}_j = [1-a_i/(1+a_{i+1})] \prod_{j=0}^{i-1} [a_j/(1+a_{j+1})]$$

for $i = 1,2,\ldots$

and

$$R_0 \equiv 1$$

$$R_i = \prod_{j=0}^{i-1} \bar{\lambda}_j = \prod_{j=0}^{i-1} [a_j/(1+a_{j+1})]$$

for $i=1,2,\ldots$

where $\{a_n\}$ is a sequence defined in Theorem 1.

To prove Theorem 1, we need the following lemma from Ebrahimi [3].

Lemma 1. Let $\{a_n\}$ and $\{b_n\}$ be two converging positive sequences and let $c > 0$. If $a_n/(c+a_{n+1}) \equiv b_n/(c+b_{n+1})$ for all n, then $a_n \equiv b_n$ for all n.

Proof. The proof of the lemma involves essentially case I: $a \equiv b \neq 0$ and case II: $a \equiv b = 0$, where $\lim_{n\to\infty} a_n \equiv a$ and $\lim_{n\to\infty} b_n \equiv b$.

Proof of Theorem 1. First assume that the life distribution is IDMRL at n_0. Recall we have assumed $m_n > 0$ for all n. Note that

$$\begin{aligned} m_n &= \sum_{j=n}^{\infty} R_{j+1}/R_n = \sum_{i=n+1}^{\infty} R_i/R_n \\ &= \sum_{i=n+1}^{\infty} \prod_{j=n}^{i-1} (R_{j+1}/R_j) \\ &= \sum_{i=n+1}^{\infty} \prod_{j=n}^{i-1} \bar{\lambda}_j \qquad \text{(A.1)} \\ &= \bar{\lambda}_n + \bar{\lambda}_n \sum_{i=n+2}^{\infty} \prod_{j=n+1}^{i-1} \bar{\lambda}_j \\ &= \bar{\lambda}_n (1 + m_{n+1}) \end{aligned}$$

holds for any MRL, m_n.

Thus, we express

$$\bar{\lambda}_n = m_n/(1 + m_{n+1}) \; . \tag{A.2}$$

By definition of IDMRL at n_0 the sequence $\{a_n\}$ defined by $a_n = m_n$ satisfies (2.5). From (A.2), $\{a_n\}$ also fulfills (2.6). Note that $a_n = m_n > 0$ for all n. This completes the first implication of Theorem 1.

To prove the implication in the other direction, we assume the existence of a sequence $\{a_n\}$ satisfying (2.5), (2.6) and $a_n > 0$ for all n. By a_n decreasing for $n \geq n_0$ and $a_n > 0$, $\lim_{n\to\infty} a_n = a \geq 0$ exists.

Now consider

$$m_n \equiv \sum_{i=n+1}^{\infty} \prod_{j=n}^{i-1} \bar{\lambda}_j \tag{A.3}$$

$$\equiv \sum_{i=1}^{\infty} \prod_{j=n}^{i-1+n} \bar{\lambda}_j$$

which follows directly from the representation of $\bar{\lambda}_j$ in (2.6), the definition of m_n, (A.1), and readjusting the summation index. Note that

$$\lim_{n\to\infty} \bar{\lambda}_n \equiv \lim_{n\to\infty} a_n/(1+a_{n+1}) = a/(1+a). \tag{A.4}$$

By (A.4) and the ratio test, m_n converges, and hence, is well defined. Consider next

$$\left.\begin{aligned} \lim_{n\to\infty} m_n/(1+m_{n+1}) &= \lim_{n\to\infty} [\bar{\lambda}_n(1+m_{n+1})]/(1+m_{n+1}) \\ &= \lim_{n\to\infty} \bar{\lambda}_n = a/(1+a) \; . \end{aligned}\right\} \tag{A.5}$$

Using limiting operations, we have

$$\lim_{n\to\infty} m_n = \sum_{i=1}^{\infty} \lim_{n\to\infty} \left(\prod_{j=n}^{i-1+n} \bar{\lambda}_j \right)$$

$$= \sum_{i=1}^{\infty} \left(\lim_{n\to\infty} \bar{\lambda}_n \right)^i = \sum_{i=1}^{\infty} \left(\frac{a}{1+a} \right)^i \equiv a \, .$$

By $\bar{\lambda}_n = a_n/(1+a_{n+1}) = m_n/(1+m_{n+1})$ and the results above, we apply Lemma 1 to see $a_n = m_n$ for all n. By property (2.5) of $\{a_n\}$, the sequence $\{m_n\}$ satisfies the condition for IDMRL at n_0, which we wanted to show.

For the DIMRL at n_0 version of Theorem 1 the proof assuming first DIMRL (n_0) follows similarly. To show the implication in the other direction, however, requires the additional condition that the sequence $\{a_n\}$ is bounded for these proof techniques. Watson & Wells [12] considered lognormal's unbounded MRL. We allow for unbounded $\{a_n\}$ and MRL by our comments and conditions on page 7. Cf. also Bhattacharjee [2] and Hall & Wellner [7].

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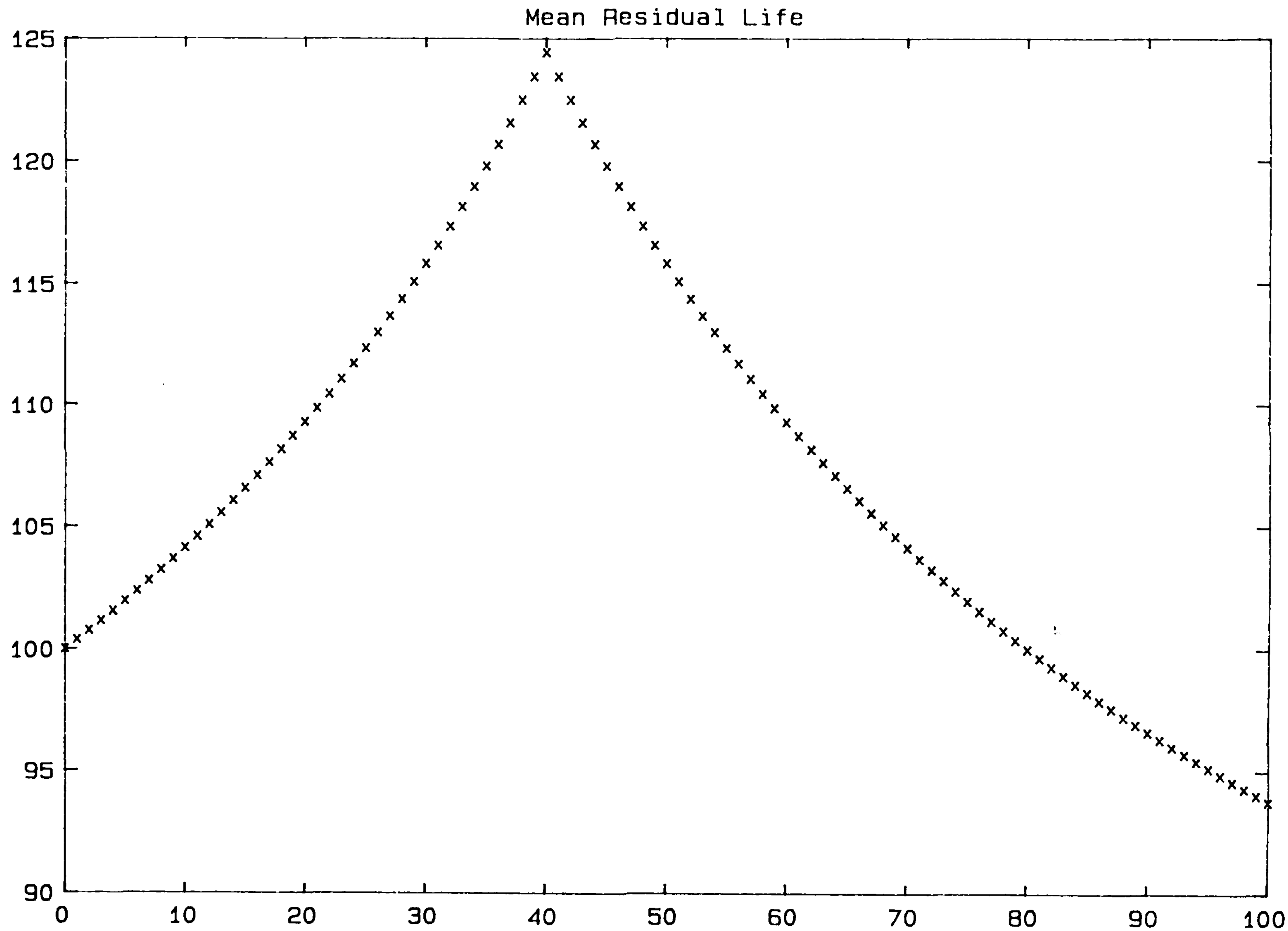
FIGURES

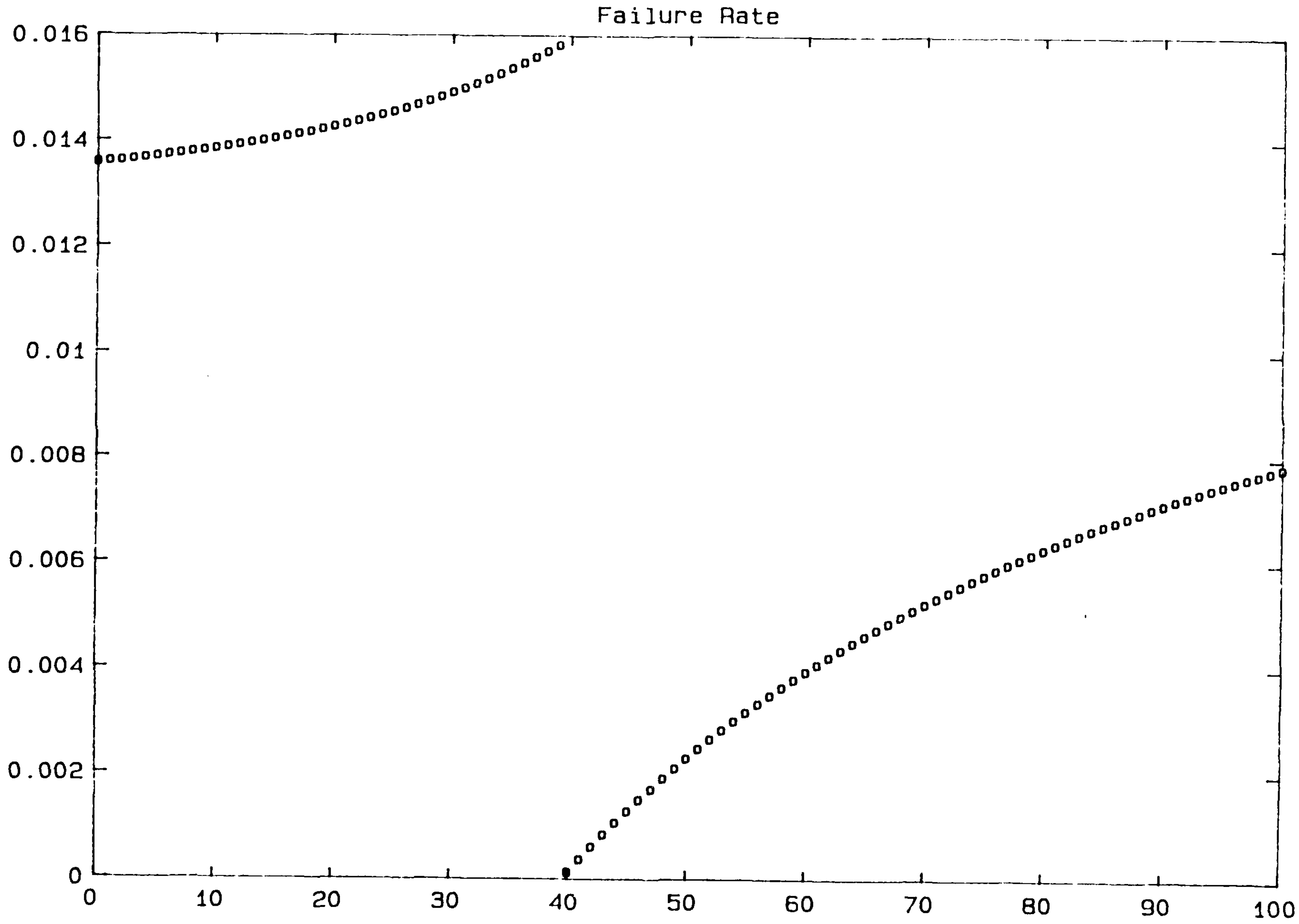
Captions

Figure 1: Mean residual life function

Figure 2: Failure rate function



Mean Residual Life
125
120
115
110
105
100
95
90
0
10
20
30
40
50
60
70
80
90
100



Failure Rate
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
10
20
30
40
50
60
70
80
90
100

END
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